STUDIES ON OPTIMAL DESIGNING OF CERTAIN SKIP-LOT SAMPLING REINSPECTION SCHEMES

A SYNOPSIS

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1. INTRODUCTION

Product quality plays an efficient role in purchasing decisions of the product and in increasing the profitability as well as reputation of the company. Product quality describes a product's capability to fulfill the standards specified by the consumers and usability, efficiency, etc. are some of the common expectations on products considered when evaluating a product's quality. One can evaluate the product quality and subsequently provide quality assurance either by employing complete inspection (or 100% inspection) or sampling inspection. However, due to time and cost consumption involved in complete inspection, this inspection can be avoided unless it is imperative for finding all nonconforming products. In this circumstance, sampling inspection is preferred for quality evaluation and assurance. Acceptance sampling is one form of sampling inspection implemented to provide quality assurance for submitted lot of products. Employing acceptance sampling in the field of production is becoming more and more popular as it is considered to offer acceptance/rejection decision with less inspection efforts.

According to Dodge [1], acceptance sampling may be categorized into four major divisions as follows:

1) Lot-by-Lot acceptance sampling by the method of attributes, in which each unit in a sample is inspected on a go-not-go basis for one or more characteristics
2) Lot-by-Lot sampling by the method of variables, in which each unit in a sample is measured for a single characteristic, such as weight or strength, etc.
3) Continuous sampling of flow of units by the method of attributes; and
4) Special purpose plans including chain sampling, skip-lot sampling, small-sample plans, etc.

Basic Concepts and Terminologies

Most of the definitions are taken from American National Standards Institute/American Society for Quality Control (ANSI/ASQC) standard A2 [2]. It prescribes an acceptance-sampling scheme as a collection of procedures that typically

consists of acceptance sampling plans in which lot sizes, sample sizes, and acceptance requirements, as well as the amount of 100% inspection and sampling are required.

Nonconformity and Nonconforming unit

A nonconformity is a departure of a quality characteristic from its intended level or state that occurs with a severity sufficient to cause an associated product or service not to meet a specification requirement. A nonconforming unit is a unit of product or service containing at least one nonconformity.

Acceptable Quality Level (AQL)

The maximum percentage or proportion of variant units in a lot or batch that, for the purposes of acceptance sampling, can be considered satisfactory as a process average is called the acceptance quality level. It is often symbolized by p_1.

Limiting Quality Level (LQL)

The percentage or proportion of variant units in a batch or lot for which, for the purposes of acceptance sampling, the consumer wishes the probability of acceptance to be restricted to a specified low value is called the limiting quality level. This is also referred to as rejectable quality level (RQL) or lot tolerance percent defective (LTPD). Percent defective is also termed as proportion non-conforming or fraction non-conforming. It is represented by p_2.

Average Outgoing Quality (AOQ)

The AOQ is the expected quality of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality.

Average Outgoing Quality Limit (AOQL)

The maximum *AOQ* over all possible levels of incoming quality is termed as AOQL. The assumption underlying in this expression is that for all accepted lots the average proportion non-conforming is assumed to be p and for all rejected lots the entire units are being screened and non-conforming units are replaced.

Average Sample Number (ASN)

Average sample number (ASN) is defined as “the average sample units per lot used for making decisions either acceptance or non acceptance”.

Average Total Inspection (ATI)

Average total inspection (ATI) is defined as “the average number of units inspected per lot based on the sample size for accepted lots and all inspected units in rejected lots”.

Average Run Length (ARL)

The average run length (ARL) of the plan detects a sudden or abrupt change in the process average or fraction non-conforming, *p*, will be evaluated by measures based on the number of lots to the occurrence of a rejection.

Operating Characteristic Curve

A graphical representation of a given probability of acceptance for various levels of incoming quality is considered an operating characteristic (OC) curve. This is one of the most significant performance measures and expresses the discriminating powers to any sampling plan. Usually on horizontal and vertical axes, the OC curve plots the probability of accepting the lot versus the lot fraction defective. To connect all possible quality levels, a smooth curve is usually drawn. In general, there are two kinds of OC curves namely, Type-A and Type-B.

Type A OC curve is used for unique lots, or a lot from an isolated sequence. "A curve showing, the probability of accepting a lot as a function of lot quality".

Type B OC curve is used for a continuous stream of lots. "A curve showing for a plan, the probability of accepting a lot as a function of the process average."

Designing of Sampling Plans

According to Peach [3], one of the most familiar approaches involved in designing acceptance sampling plans is as follows.

The plan is specified by requiring the OC curve to pass through two specified points. The two points generally selected are (p_1, 1-α) and (p_2, β), where p_1 is the PQL, p_2 is the CQL, α is the producer's risk and β is the consumer's risk. Tables of Cameron [4] are the instances of this type of designing. Usually, the values of α and β are considered to be 5% and 10%, respectively.

Markov Chain

A stochastic process $\{X_n, n \in N\}$ with discrete state space *S* and with discrete parameter space *N* is called Markov chain provided

$$P(X_{n+1} = j \mid X_0, ..., X_n = i) = P(X_{n+1} = j \mid X_n = i) = p_{ij}$$

for all $i, j \in S, n \in N$. p_{ij} 's are called the transition probabilities and they form a square matrix P called the transition probability matrix. Next,

$$p_{ij}(m,n) = P(X_n = j \mid X_m = i), 0 \le m \le n$$

denotes the probability that the chain makes a transition from state *i* at time step *m* to state *j* at time step *n*. A Markov chain is said to be homogeneous if $p_{ij}(m, n)$ depends

only on the difference (n − m), otherwise the chain is a non homogeneous Markov chain.

2. LITERATURE REVIEW

One of the major areas for acceptance sampling classified under the fourth category of classification of Dodge [1] is the special purpose inspection plans, often known as special purpose plans. They are specialized for certain purposes, as contrasted to general or universal usage. In order to inspect a series of products with minimum effort, Dodge [5] applied the principles of continuous sampling plans and introduced a new sampling plan namely skiplot sampling plan (SkSP). In general, the designing of SkSPs is motivated for reducing inspection cost. SkSPs require smaller ASN for inspection as compared to the single sampling plan (SSP). The use of SkSP is efficacious in foreshortening the effort and inspection's time of the product with a history of excellent quality.

Perry [6] has developed a procedure for designing SkSP-2, and has shown that this plan is advantageous for small sample situations and for costly and destructive testing. To know more details on skip-lot sampling concept, one may refer to Perry [7], Parker and Kessler [8], Balamurali and Subramani [9, 10, 11], Balamurali and Jun [12], Aslam et al. [13, 14, 15, 16] and Vijayaragavan [17].

Govindaraju and Ganesalingam [18] developed the concept of resampling in which the lot rejected in the original inspection is permitted to resubmit the sample for inspection and the performance measures were derived with SSP as a reference plan. Although resampling scheme considers additional samples from the same lot as in double sampling plan (DSP), there is a difference between the sampling procedures of them. That is, in DSP, the decision is made by combining the information from the first sample as well as second sample in case the second sample is taken. However, the result of the first sample inspection is discarded in resampling scheme.

Several investigations to highlight the performance of resampling scheme have been carried out and can be seen in the literature. Aslam et al. [19] developed a sampling plan under gamma-Poisson distribution for attribute inspection. Resampling scheme for assuring variables quality characteristics was given by Balamurali and Usha [20]. Balamurali [21] proposed an attri-vari inspection policy for the resubmitted lots based on the process capability index C_{pk} .To know more details on resampling concept, one may refer to Aslam et al. [22], Balamurali et al. [23], Khan et al. [24] and Tanveer et al. [25].

Balamurali et al. [26] first incorporated the resampling scheme in SkSP and proposed an innovative sampling plan namely, SkSP-R. Balamurali and Usha [27] proposed SkSP-2 plan for resubmitted lots and the resultant plan is designated as SkSP-2-R. They exemplified that SkSP-2-R plan provides an outstanding performance than SkSP-2 plan and SkSP-R plan. Aslam et al. [28] proposed a group skip-lot sampling plan using resampling (SkSP-R) for accelerated life tests by assuming that the lifetime of a product follows Weibull distribution with known shape parameter. The economic design for SkSP-R sampling plan was investigated by Balamurali et al. [29]. Hussain et al. [30] presented the designing methodology for SkSP-R plan based on process capability index C_{pk}.

3. OBJECTIVES

The main objectives of this research work are

- To Propose a sampling plan by assimilating the concept of resampling in SkSP-2 with double sampling plan as a reference plan and designated the new plan as SkDSP-2-R.
- To Develop a sampling plan by incorporating the idea of resampling in SkSP-3 and designated the new plan as SkSP-3-R.
- To Develop a sampling plan by incorporating the idea of resampling in two-level skip lot sampling plan SkSP-2L.1 and designated the new plan as SkSP-2L.1-R.
- To Develop a sampling plan by integrating the idea of resampling in two-level skip lot sampling plan SkSP-2L.2 and designated the new plan as SkSP-2L.2-R.
- To Propose a sampling plan by incorporating the idea of resampling in two-level skip lot sampling plan SkSP-2L.3 and designated the new plan as SkSP-2L.3-R.

4. RESEARCH PROBLEM

The main focus of our research problem is, to incorporate the concept of resampling scheme in Skip-lot sampling SkSP-2-R with double samping plan as a reference plan, skip-lot sampling SkSP-3, and three two-level skip-lot sampling plans namely, SkSP-2L.1, SkSP-2L.2 and SkSP-2L.3.

4.1 Evaluation and Optimal Designing of a Skip-Lot Sampling Reinspection Plan SkSP-2-R with Double Sampling Plan as a Reference Plan

SkSP-2-R plan with DSP as a reference plan

Consider SkSP-2-R with DSP as the reference plan and the plan is designated as SkDSP-2-R. The operating procedure of SkDSP-2-R plan is stated as follows (see Balamurali and Usha [27]):

(1) Start with normal inspection using DSP as the reference plan. During normal inspection, lots are inspected one by one in the order of being submitted.

(2) After the acceptance of i consecutive lots under normal inspection, switch to skipping inspection.

(3) During the skipping inspection, inspect only a fraction f of the lots which selected in random manner. Continue the skipping inspection up to rejection of the sampled lot.

(4) After the rejection of the lot; go for resampling procedure to the immediate next lot as in (5) given below.

(5) During resampling procedure, inspect the lot using the reference plan. If the lot is accepted, then continue the skipping inspection. On non-acceptance of the lot, carry out the resampling for m times and reject the lot if it has not been accepted on $(m\text{-}1)^{st}$ resubmission.

(6) If a lot is rejected on resampling scheme, then revert to the normal inspection promptly as in (1) otherwise continue the skipping inspection as in (3).

(7) Replace all the non-conforming units by conforming units in the rejected lots.

The SkDSP-2-R plan is absolutely characterized by seven parameters, which are n_1, n_2, c_1, c_2, i, f $(0 < f < 1)$, and m.

Designing Methodology of SkDSP-2-R Plan

According to Balamurali and Usha [27], the operating characteristic (OC) function of SkDSP-2-R plan is stated as follows:

$$P_a(p) = \frac{fQ^m(1-P^i) + P^i - fQ^{m+1}}{fQ^m(1-P^i) + P^i(1+fQ)}$$

where $P_a(p)$ is the lot acceptance probability under proposed SkDSP-2-R plan, and P is the lot acceptance probability under DSP and $Q = 1 - P$. Now, P and Q are determined as

$$P=\sum_{i=0}^{c_1}\binom{n_1}{i}p^i(1-p)^{n_1-i}+\sum_{i=c_1+1}^{c_2}\binom{n_1}{i}p^i(1-p)^{n_1-i}\left[\sum_{j=0}^{c_2-i}\binom{n_2}{j}p^j(1-p)^{n_2-j}\right] \quad (4.1)$$

For stated PQL (= p_1)) and CQL (= p_2), Equation (4.1) is given by

$$P_1=\sum_{i=0}^{c_1}\binom{n_1}{i}p_1^i(1-p_1)^{n_1-i}+\sum_{i=c_1+1}^{c_2}\binom{n_1}{i}p_1^i(1-p_1)^{n_1-i}\left[\sum_{j=0}^{c_2-i}\binom{n_2}{j}p_1^j(1-p_1)^{n_2-j}\right]$$

$$P_2=\sum_{i=0}^{c_1}\binom{n_1}{i}p_2^i(1-p_2)^{n_1-i}+\sum_{i=c_1+1}^{c_2}\binom{n_1}{i}p_2^i(1-p_2)^{n_1-i}\left[\sum_{j=0}^{c_2-i}\binom{n_2}{j}p_2^j(1-p_2)^{n_2-j}\right]$$

$$Q_1=1-P_1 \text{ and } Q_2=1-P_2.$$

The ASN of the SkDSP-2-R plan is given by

$$ASN(p)=(ASN_{double}(p))\frac{f[Q^m(1-P^i)+P^i(1+Q)]}{fQ^m(1-P^i)+P^i(1+fQ)}$$

where $ASN_{double}(p)$ is the ASN of DSP, which is given by

$$ASN_{double}(p)=n_1+n_2\sum_{i=c_1+1}^{c_2}\binom{n_1}{i}p^i(1-p)^{n_1-i}$$

Thus, the optimal parameters of the proposed SkDSP-2-R plan are determined by using the optimization problem as given below:

$$\textit{Minimize}\quad ASN(p_1)=\left(n_1+n_2\sum_{i=c_1+1}^{c_2}\binom{n_1}{i}p_1^i(1-p_1)^{n_1-i}\right)\left(\frac{f[Q_1^m(1-P_1^i)+P_1^i(1+Q_1)]}{fQ_1^m(1-P_1^i)+P_1^i(1+fQ_1)}\right)$$

$$\textit{Subject to}\quad P_a(p_1)=\frac{fQ_1^m(1-P_1^i)+P_1^i-fQ_1^{m+1}}{fQ_1^m(1-P_1^i)+P_1^i(1+fQ_1)}\geq 1-\alpha$$

$$P_a(p_2)=\frac{fQ_2^m(1-P_2^i)+P_2^i-fQ_2^{m+1}}{fQ_2^m(1-P_2^i)+P_2^i(1+fQ_2)}\leq \beta$$

$$n_1>1,\ n_2>1,\ c_2>c_1\geq 0,\ i\geq 1,\ 0<f<1,\ m>1$$

The optimal parameters of SkDSP-2-R plan have been determined by formulating the optimization problem where the minimization of ASN is an objective function and the constraints are probabilities of acceptance at PQL and CQL along with respective producer and consumer risks. The implementation of SkDSP-2-R plan is illustrated by an example. The advantages of the proposed SkDSP-2-R plan over SkSP-2-R plan, SkDSP-2 plan and DSP have been discussed in terms of OC, ASN and ATI. The SkDSP-2-R plan gives better protection to producer as well as the consumer.

4.2 Evaluation and Optimal Designing of a Skip-Lot Sampling SkSP-3 Including Reinspection Scheme

Executing procedure of SkSP-3-R plan

The executing procedure of the proposed SkSP-3-R plan with SSP as a reference plan is exemplified as follows:

(1) Start with normal inspection by using SSP as the reference plan and in which each lot is inspected.

(2) After the acceptance of i successive lots in normal inspection, switch to skipping inspection.

(3) During the skipping inspection, inspect only a fraction f of the lots selected randomly. Continue the skipping inspection until a sampled lot is rejected.

(4) If the lot rejection occurs on skipping inspection, then inspect the next k submitted lots.

(5) If there will be any rejection in inspecting k lots, then switch to normal inspection as in (1).

(6) If all the k lots are acceptable, continue the skipping inspection as in (3).

(7) If the lot rejection occurs at the k^{th} lot, move on to the resampling procedure to the instantaneous next lot as in (8).

(8) In the resampling procedure, the inspection is carried out using the reference plan SSP. On non-acceptance of the lot, carry out the resampling procedure for m times and reject the lot if it has not been accepted on $(m\text{-}1)^{st}$ resubmission.

(9) Revert to the normal inspection as in (1) if the lot rejection occurs on the resampling scheme.

(10) Revert to the skipping inspection as in (3) if the lot acceptance occurs on the resampling scheme.

(11) Replace all the nonconforming units with conforming units in the rejected lots.

The SkSP-3-R plan is characterized by five parameters namely, f, i, k, m, n, and c.

Performance measures of SkSP-3-R plan

The performance measures are derived by using the Markov chain formulation. We contemplate the following measures of interest.

1) The OC function or probability of acceptance ($P_a(p)$) of the SkSP-3-R plan is

$$P_a(p) = \frac{f\left(P - P^i\right)\left(1 - P^{k-1}\left(1 - Q^{m+1}\right)\right) + P^i\left(2 - P^k - fQ\right)}{f\left(1 - P^i\right)\left(1 - P^{k-1}\left(1 - Q^{m+1}\right)\right) + P^i\left(2 - P^k + fQ^2P^{k-1}\right)}$$

where P is the probability of acceptance of the lot under reference plan, and $Q = 1 - P$.

2) The ASN of the SkSP-3-R plan is

$$ASN(p) = \frac{n\left(f\left(1-P^i\right)\left(1-P^{k-1}\left(1-Q^{m+1}\right)\right)+ fP^i\left(2-P^k+Q^2P^{k-1}\right)\right)}{f\left(1-P^i\right)\left(1-P^{k-1}\left(1-Q^{m+1}\right)\right)+P^i\left(2-P^k+fQ^2P^{k-1}\right)}$$

3) The ATI of the SkSP-3-R plan is

$$ATI(p) = \frac{\left(n+(N-n)(1-P)\right)\left(f\left(1-P^i\right)\left(1-P^{k-1}\left(1-Q^{m+1}\right)\right)+ fP^i\left(2-P^k+Q^2P^{k-1}\right)\right)}{f\left(1-P^i\right)\left(1-P^{k-1}\left(1-Q^{m+1}\right)\right)+P^i\left(2-P^k+fQ^2P^{k-1}\right)}$$

Characteristics of response-to-change of SkSP-3-R plan

Perry [6] used average run length (ARL) measures such as ARL-1 and ARL-2 to evaluate the characteristics of response-to-change of skip-lot sampling plans and the measures are defined as follows:

ARL-1: The expected number of lots to the incidence of a rejection following the process quality changes abruptly from p_0 to p_1, and the count begins on the first lot following the change (the lots are counted in both skipped and inspected).

ARL-2: The expected number of lots to the incidence of a rejection following rejection (the count commences at the start of a normal inspection).

Perry [6] utilized the general formula for finding ARL measures ARL-1 and ARL-2. Similarly, the formula for ARL of the SkSP-3-R plan is also derived. The formula for ARL-1 of the proposed SkSP-3-R plan is derived as follows:

$$ARL\text{-}1 = 1 + \sum_{j=1}^{\infty} P(A_1, A_2, ...A_j | N_0)P(N_0) + ... + \sum_{j=1}^{\infty} P(A_1, A_2, ...A_j | SAR)P(SAR) \tag{4.2}$$

where P (A_1, A_2, …., A_j) symbolizes the probability of j lots successively accepted immediately after the count begins and $P(.)$ symbolizes the long run probabilities. The joint probabilities will be determined by first conditioning upon the appropriate states of the Markov chain, multiplying them by long-run probabilities of the state and summing over all the states.

The ARL-2 of SkSP-3-R will be determined as

$$ARL\text{-}2 = \frac{1}{1-P_a(p)} \tag{4.3}$$

where $P_a(p)$ is the acceptance probability of the SkSP-3-R plan given in (1).

After simplifying the ARL-1 formula provided in (4.2), we get

$$\text{ARL -1} = \frac{1}{Q_1 D_0}\left\{\begin{array}{l} Q_1 D_0 + fP_1\left(1-P_0^{k-1}\left(1-Q_0^{m+1}\right)\right)\left(1-P_0^{i+1}\right)+(1-f)\left(1-P_0^{k-1}\left(1-Q_0^{m+1}\right)\right)\sum_{j=0}^{i} P_0^j P_1^{i-j} + \\ \frac{(1-fQ_1)}{f} P_0^i\left[\left(1-P_0^k\right)(1-fQ_0)+ fQ_0^2 P_0^{k-1}\left(2-Q_0^m-Q_1^m\right)+1\right] \end{array}\right\}$$

where $D_0 = f(1-P_0^i)(1-P_0^{k-1}(1-Q_0^{m+1})) + P_0^i(2-P_0^k + fQ_0^2 P_0^{k-1})$, P_0 and P_1 represent the acceptance probabilities of the lot under the reference plan at process quality levels p_0 and p_1 respectively and Q_0=1 – P_0 and Q_1=1– P_1.

The ARL-2 of the proposed SkSP-3-R plan can be determined as

$$ARL-2 = \frac{f\left(1-P_1^i\right)\left(1-P_1^{k-1}\left(1-Q_1^{m+1}\right)\right)+P_1^i\left(2-P_1^k + fQ_1^2 P_1^{k-1}\right)}{fQ_1\left(1-P_1^{k-1}\left(1-Q_1^{m+1}\right)+P_1^i + Q_1 P_1^{i+k-1}\right)}$$

The optimization technique is used to determine the plan parameters for various quality levels with their specified risks. An illustration is given to describe the implementation of the SkSP-3-R plan. The efficiency of the proposed SkSP-3-R plan is emphasized by comparing the SkSP-3-R plan with the SkSP-3 plan, SkSP-2-R plan, SkSP-2 plan, and SSP through OC, ASN, and ATI. The results show that the implementation of the SkSP-3-R plan requires minimum inspection effort to provide simultaneous protection for both producer and consumer and the SkSP-3-R plan has superior ability in detecting the process variation rather than other plans.

4.3 Evaluation and Optimal Designing of Two-Level Skip-Lot Sampling SkSP-2L.1 Plan for Resubmitted Lots

Operating Procedure of SkSP-2L.1-R plan

The SkSP-2L.1-R plan is an innovative plan, developed on the basis of principles of two-level SkSP-2L.1 and resampling scheme in order to inspect the quality of continuous flow of bulk products. As in SkSP-2 plan of Perry [6], the proposed plan SkSP-2L.1-R plan makes the use of the reference plan. The following sampling procedure describes the implementation of the proposed SkSP-2L.1-R plan.

(1) Begin with normal inspection using the reference plan in which every single lot is inspected.

(2) After the acceptance of i consecutive lots under normal inspection, switch to the first level skipping inspection.

(3) In the first level skipping inspection, lots are inspected at a rate of f_1. After the acceptance of k lots in succession in the first level skipping inspection, switch

to the second level skipping inspection as in (4) below. When a lot is rejected before *k* successive lots, switch to the normal inspection as in (1) above.

(4) Replace all the nonconforming units by conforming units in the rejected lots.

(5) In the second level skipping inspection, lots are inspected at a rate of f_2 ($< f_1$). Continue the second level skipping inspection up to the rejection of a lot. After the rejection of the lot, go for resampling to the immediate next lot as in (6).

(6) In the resampling procedure, carry out the inspection by means of the reference plan. On non-acceptance of the lot, resampling is done for *m* times, and the lot is rejected if it has not been accepted on $(m\text{-}1)^{st}$ resubmission.

(7) Revert to the first level skipping inspection as in (3) if a lot is rejected on resampling scheme.

(8) Revert to the second level skipping inspection as in (5) if a lot is accepted on resampling scheme.

Performance Measures of SkSP-2L.1-R Plan

Using the Markov chain formulation, the performance measures are derived and given as follows:

1) The OC function or probability of acceptance of the lot under SkSP-2L.1-R plan is

$$P_a(p) = \frac{f_1 f_2 Q^m (P - P^i)(1 - P^k) + f_2 Q^m P^i (1 - P^k) + f_1 P^{i+k}}{f_1 f_2 Q^m (1 - P^i)(1 - P^k) + f_2 Q^m P^i (1 - P^k) + f_1 P^{i+k} + f_1 f_2 P^{i+k} Q}$$

where P is the probability of acceptance of the lot under reference plan, and $Q = 1 - P$.

2) The ASN of the proposed plan is

$$ASN(p) = \frac{n(f_1 f_2 Q^m (1 - P^i) + f_1 f_2 P^{i+k} Q + f_1 f_2 P^{i+k} + f_1 f_2 Q^m P^i (1 - P^k))}{f_1 f_2 Q^m (1 - P^i)(1 - P^k) + f_2 Q^m P^i (1 - P^k) + f_1 P^{i+k} + f_1 f_2 P^{i+k} Q}$$

3) The ATI of the proposed plan is

$$ATI(p) = \frac{(n + (N - n)(1 - P))(f_1 f_2 Q^m (1 - P^i) + f_1 f_2 P^{i+k} Q + f_1 f_2 P^{i+k} + f_1 f_2 Q^m P^i (1 - P^k))}{f_1 f_2 Q^m (1 - P^i)(1 - P^k) + f_2 Q^m P^i (1 - P^k) + f_1 P^{i+k} + f_1 f_2 P^{i+k} Q}$$

The value of P, probability of acceptance of the lot under reference plan, is calculated using binomial distribution and given in the following equation.

$$P = \sum_{d=0}^{c} \binom{n}{d} p^d (1 - p)^{n-d}$$

For specified p_1 (or PQL) and p_2 (or CQL), the probabilities of acceptance of the lot under reference plan are given by

$$P_1 = \sum_{d=0}^{c} \binom{n}{d} p_1^d (1-p_1)^{n-d} \text{ and } P_2 = \sum_{d=0}^{c} \binom{n}{d} p_2^d (1-p_2)^{n-d}$$

Thus, the following optimization problem is involved in determining the optimal parameters of proposed SkSP-2L.1-R plan.

$$\textit{Minimize } ASN(p_1) = \frac{n(f_1 f_2 Q_1^m (1-P_1^i) + f_1 f_2 P_1^{i+k} Q_1 + f_1 f_2 P_1^{i+k} + f_1 f_2 Q_1^m P_1^i (1-P_1^k))}{f_1 f_2 Q_1^m (1-P_1^i)(1-P_1^k) + f_2 Q_1^m P_1^i (1-P_1^k) + f_1 P_1^{i+k} + f_1 f_2 P_1^{i+k} Q_1}$$

Subject to

$$P_a(p_1) = \frac{f_1 f_2 Q_1^m (P_1 - P_1^i)(1-P_1^k) + f_2 Q_1^m P_1^i (1-P_1^k) + f_1 P_1^{i+k}}{f_1 f_2 Q_1^m (1-P_1^i)(1-P_1^k) + f_2 Q_1^m P_1^i (1-P_1^k) + f_1 P_1^{i+k} + f_1 f_2 P_1^{i+k} Q_1} \geq 1-\alpha$$

$$P_a(p_2) = \frac{f_1 f_2 Q_2^m (P_2 - P_2^i)(1-P_2^k) + f_2 Q_2^m P_2^i (1-P_2^k) + f_1 P_2^{i+k}}{f_1 f_2 Q_2^m (1-P_2^i)(1-P_2^k) + f_2 Q_2^m P_2^i (1-P_2^k) + f_1 P_2^{i+k} + f_1 f_2 P_2^{i+k} Q_2} \leq \beta$$

The proposed plan consists of five parameters, namely, f_1, f_2 ($0 < f_1, f_2 < 1$ and $f_1 > f_2$) as well as reference plan, where f_1 and f_2 represent the fraction of lots to be inspected in the first and second level skipping inspections respectively, i and k are the respective clearance numbers of normal and skipping inspections, m, the number of times the lots are resubmitted for inspection, and i, k, m are the positive integers with $k \leq i$.

The plan parameters of the proposed SkSP-2L.1-R plan have been determined in such a way that the ASN has been minimized subject to satisfying the PQL and CQL with respective producer and consumer risks. The implementation of the proposed plan SkSP-2L.1-R has been given by an illustrative example. Through the curves of OC, ASN and ATI, the advantages of the SkSP-2L.1-R plan over SkSP-2-R plan, SkSP-2 plan and SSP have been discussed. The SkSP-2L.1-R plan affords safety to the producer and safeguards the consumer than SSP, SkSP-2 plan and SkSP-2-R plan with minimum inspection. It has been observed that the SkSP-2L.1-R plan uses smaller ASN and ATI for the inspection purpose than SSP, SkSP-2 plan and SkSP-2-R plan.

4.4 Evaluation and Optimal Designing of Two-Level Skip-Lot Sampling SkSP-2L.2 Including Reinspection Scheme

SkSP-2L.2-R plan

The implementation of the proposed SkSP-2L.2-R plan is portrayed by the sampling procedure as given below:

(1) Start with normal inspection using the reference plan in which the lots are inspected one by one in the order of being submitted.

(2) Switch to the first level skipping inspection if i consecutive lots are accepted under normal inspection.

(3) During the first level skipping inspection, inspect lots at a rate of f_1. After the acceptance of k successive lots in the first level skipping inspection, switch to the second level skipping inspection as in (4) below. When a lot is rejected before accepting k successive lots, switch to the normal inspection as in (1) above.

(4) During second level skipping inspection, inspect lots at a rate of f_2. Continue the second level skipping inspection until the sampled lot is rejected. After the rejection of the lot, go for resampling to the immediate next lot as in (5).

(5) During the resampling procedure, inspect the lot by utilizing the reference plan. If the lot is accepted, then continue the second level skipping inspection. Otherwise, carry out resampling for m times on non-acceptance of the lot and reject the lot if it has not been accepted on $(m\text{-}1)^{st}$ resubmission.

(6) During the resampling procedure, if a lot is rejected, go for normal inspection as in (1).

(7) Replace all the nonconforming units by conforming units in the rejected lots.

The SkSP-2L.2-R plan is characterized by seven parameters namely, f_1, f_2, i, k, m, n, and c.

Performance Measures of SkSP-2L.2-R Plan

The performance measures of SkSP-2L.2-R plan are derived by using the limiting probabilities of the Markov chain and the measures are as follows:

1) The OC function or probability of acceptance of the lot under SkSP-2L.2 -R plan is

$$P_a(p) = \frac{f_1 f_2 Q^m (P - P^i) + f_2 Q^m P^i (1 - P^k) + f_1 P^{i+k}}{f_1 f_2 Q^m (1 - P^i) + f_2 Q^m P^i (1 - P^k) + f_1 P^{i+k} + f_1 f_2 P^{i+k} Q}$$

where P is the probability of acceptance of the lot under reference plan, and $Q = 1 - P$.

2) The ASN of the proposed plan is

$$ASN(p) = \frac{n\left(f_1 f_2 Q^m \left(1 - P^{i+k}\right) + f_1 f_2 P^{i+k}\left(1+Q\right)\right)}{f_1 f_2 Q^m \left(1 - P^{i}\right) + f_2 Q^m P^i \left(1 - P^{k}\right) + f_1 P^{i+k} + f_1 f_2 P^{i+k} Q}$$

3) The ATI of the proposed plan is

$$ATI(p) = \frac{\left(n + \left(N - n\right)\left(1 - P\right)\right)\left(f_1 f_2 Q^m \left(1 - P^{i+k}\right) + f_1 f_2 P^{i+k}\left(1+Q\right)\right)}{f_1 f_2 Q^m \left(1 - P^{i}\right) + f_2 Q^m P^i \left(1 - P^{k}\right) + f_1 P^{i+k} + f_1 f_2 P^{i+k} Q}$$

Using the optimization problem, the plan parameters of the proposed SkSP-2L.2-R plan are determined. An illustrative example describes the implementation of the proposed plan SkSP-2L.2-R. The advantages of the SkSP-2L.2-R plan over SkSP-2L.2 plan, SkSP-2 plan and SSP are discussed through the curves of OC, ASN and ATI. The SkSP-2L.2-R plan defends the producer and safeguards the consumer than SSP, SkSP-2 plan, and SkSP-2L.2 plan. It is observed that the SkSP-2L.2-R plan uses the reasonable ASN and ATI for the inspection purpose based on the quality levels rather than SSP, SkSP-2 plan, and SkSP-2L.2 plan.

4.5 Evaluation and Optimal Designing of Two-Level Skip-Lot Sampling Plan SkSP-2L.3 for Resubmitted Lots

Executing procedure

The executing procedure of the SkSP-2L.3-R plan is described as follows:

(1) At the outset, begin with normal inspection using the reference plan and inspect the lots one by one in the order of production.

(2) Note when i consecutive lots are found to be acceptable

   a) If this happens on the i^{th} lot (first i lots are found to be acceptable), then switch to skipping inspection at level 2 in which the lots are inspected at a rate of f_2.

   b) If this happens after the i^{th} lot, then switch to skipping inspection at level 1 in which the lots are inspected at a rate of f_1.

(3) If a lot is found to be rejected on skipping inspection at level 1, then immediately revert to normal inspection as in (1).

(4) Replace all the nonconforming units by conforming units in the rejected lots.

(5) If a lot is found to be rejected on skipping inspection at level 2, go for resampling procedure to the immediate next lot as in (6).

(6) During the resampling procedure, inspect the lot by applying the reference plan. If the lot is found to be acceptable, then continue the second level skipping inspection. Otherwise, carry out resampling for m times on non-acceptance of the lot and reject the lot if it has not been accepted on (m-1)st resubmission.

(7) If a lot is found to be rejected in the resampling procedure, go for normal inspection as in (1).

The SkSP-2L.3-R plan is characterized by seven parameters namely, f_1, f_2, i, m, n and c.

Performance measures of SkSP-2L.3-R plan

The performance measures are derived by using Markov chain approach which are as follows:

1) The OC function or probability of acceptance of the lot under SkSP-2L.3-R plan is

$$P_a(p) = \frac{f_1 f_2 Q^m \left(P - P^i\right) + f_2 Q^m P^i \left(1 - P^i\right) + f_1 P^{2i}}{f_1 f_2 Q^m \left(1 - P^i\right) + f_2 Q^m P^i \left(1 - P^i\right) + f_1 P^{2i} + f_1 f_2 P^{2i} Q}$$

where P is the probability of acceptance of the lot under reference plan, and $Q = 1 - P$.

2) The ASN of the SkSP-2L.3-R plan is

$$ASN(p) = \frac{n(f_1 f_2 Q^m (1 - P^i) + f_1 f_2 Q^m P^i (1 - P^i) + f_1 f_2 P^{2i} + f_1 f_2 P^{2i} Q)}{f_1 f_2 Q^m \left(1 - P^i\right) + f_2 Q^m P^i \left(1 - P^i\right) + f_1 P^{2i} + f_1 f_2 P^{2i} Q}$$

3) The ATI of the SkSP-2L.3-R plan is

$$ATI(p) = \frac{\left(n + (N - n)(1 - P)\right)(f_1 f_2 Q^m (1 - P^i) + f_1 f_2 Q^m P^i (1 - P^i) + f_1 f_2 P^{2i} + f_1 f_2 P^{2i} Q)}{f_1 f_2 Q^m \left(1 - P^i\right) + f_2 Q^m P^i \left(1 - P^i\right) + f_1 P^{2i} + f_1 f_2 P^{2i} Q}$$

The optimization program is developed for determining the plan parameters. In addition, the plan parameters under certain selected quality and risk conditions are determined and an illustrative example is provided to show how the proposed plan can be established and applied in the industries for practical application. The proposed SkSP-2L.3-R plan's performance and efficiency are analyzed and compared with those of the SkSP-2-R plan, SkSP-2 plan and SSP in terms of the OC, ASN and ATI.

5. ORGANIZATION OF THESIS

The thesis comprises of seven chapters.

Chapter 1, "Introduction" provides the fundamental concepts of Quality, Statistical Quality Control, Acceptance Sampling, Markov Chain and general literature review on acceptance sampling which are related to this thesis are given. This chapter consists of five sections. In the first section, the basic concepts of quality, quality dimensions and factors influencing quality are presented. In the second section, the fundamental aspects of Statistical Quality Control are also provided. In the third section, the fundamental concepts of acceptance sampling, as well as the various kinds of sampling plans and the design of sampling plans are deliberated. Some of the basic concepts of Markov chain are provided in section 4. The fifth section affords an extensive overview of the literature on attribute sampling plans and special purpose sampling plans.

Chapter 2, "Evaluation and Optimal Designing of a Skip-Lot Sampling Reinspection Plan SkSP-2-R with Double Sampling Plan as a Reference Plan" gives the designing methodology to determine the optimal parameters of skip-lot sampling plan of type 2 with double sampling plan as a reference plan for resubmitted lots designated as SkSP-2-R by using two points on the operating characteristic curve approach. Tables are constructed for various combinations of producer quality level and consumer quality level along with different producer and consumer risks. The execution of the proposed plan is provided with an illustrative example.

Chapter 3, "Evaluation and Optimal Designing of a Skip-Lot Sampling SkSP-3 Including Reinspection Scheme" presents a sampling plan by integrating the concept of resampling in a skip-lot sampling plan of type SkSP-3 and designating the resultant plan as an SkSP-3-R plan with a single sampling plan (SSP) as the reference plan. The performance measures are derived by using the Markov chain formulation. The two points on the operating characteristic curve approach is a designing methodology used to determine the plan parameters of an SkSP-3-R plan involving minimum the average sample number. Based on average run length, the SkSP-3-R plan's characteristic of response-to-change in quality is also investigated.

Chapter 4, "Evaluation and Optimal Designing of Two-Level Skip-Lot Sampling SkSP-2L.1 Plan for Resubmitted Lots" affords a sampling plan by incorporating the idea of resampling in two-level skip lot sampling plan of type SkSP-2L.1 and the new plan is designated as SkSP-2L.1-R. It provides the Markov

chain formulation of the proposed plan along with the derivation of performance measures of the plan. An industrial application of the proposed SkSP-2L.1-R plan is discussed. The efficiency of the proposed SkSP-2L.1-R with SSP as a reference plan is shown by a comparative study.

Chapter 5, "Evaluation and Optimal Designing of Two-Level Skip-Lot Sampling SkSP-2L.2 Including Reinspection Scheme" presents a sampling plan by assimilating the concept of resampling with a two-level skip lot sampling plan of type SkSP-2L.2, the new plan is designated as SkSP-2L.2-R and SSP is used as the reference plan. The performance measures of the proposed plan are derived using the Markov chain formulation. Execution of the proposed SkSP-2L.2-R plan is discussed with an illustrative example. It is proved by the comparative study that the proposed SkSP-2L.2-R plan has better performance in discriminating the lots with minimum inspection effort when compared to the existing sampling plans.

Chapter 6, "Evaluation and Optimal Designing of Two-Level Skip-Lot Sampling Plan SkSP-2L.3 for Resubmitted Lots" attempts to propose a sampling plan by assimilating the concept of resampling scheme in two-level skip-lot sampling plan of type SkSP-2L.3 and the resultant plan is designated as SkSP-2L.3-R with SSP as the reference plan. Using Markov chain formulation, the performance measures are derived. The designing methodology is provided in order to determine the plan parameters of SkSP-2L.3-R plan with the objective of minimizing the ASN by using two points on the OC curve approach. An illustrative example is provided to depict the execution of the proposed SkSP-2L.3-R plan. The comparative study substantiate that the relative efficiency of the proposed SkSP-2L.3-R plan has outstanding performance than some other existing plans under binomial and Poisson distributions.

Chapter 7, Concludes by summarizing the research work done and provides discussions for future work in this area.

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LIST OF PUBLICATIONS

1. N. Murugeswari, P. Jeyadurga and S. Balamurali, Optimal Designing of Two Level Skip-Lot Sampling Reinspection Plan, *Journal of Applied Statistics*, (2020), **49** (5), 1086-1104.
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